Heat transfer across rough surfaces

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It is argued that the heat transfer between a roughened surface and a stream of incompressible fluid flowing over it is dependent on both the viscosity and thermal conductivity of the fluid even when the roughness is large enough for viscosity to have ceased to affect the skin friction.

Concentrating on closely spaced roughness, sufficiently large for the skin friction to be independent of Reynolds number, a simple model is constructed of the flow near the surface. It consists of horseshoe eddies which wrap themselves round the individual excrescences and trail unsteadily downstream; the eddies are imagined to scour the surface and thereby to transport heat between the surface and the more vigorous flow in the neighbourhood of the roughness crests. Taken in conjunction with Reynolds analogy between temperature and velocity distributions in the fluid away from the surface, the model leads to an expression for the rate of heat transfer which contains a function of the roughness Reynolds number and the Prandtl number of the fluid whose detailed form is found by appeal to the limited experimental data available. An order-of-magnitude argument suggests that the functional form established empirically is consistent with the assumed model of the flow close to the surface.

The object of the work is to establish a basis for the analysis of experimental data and for their extrapolation with respect to Reynolds number and Prandtl number.

1. Introduction

The increase in the rate of heat transfer caused by roughening a surface exposed to a turbulent stream of fluid is less than the corresponding increase in skin friction, a fact which has been long established by experiment and one that takes little ingenuity to explain in broad terms. For the rate of heat transport in the immediate neighbourhood of the surface, no matter how irregular it might be, is controlled by a purely molecular property of the fluid, its thermal conductivity, whereas the shear stress, augmented by the roughness, is transmitted to the surface as a form drag on the individual asperities. To put the matter in a slightly different way, however much the heat transfer *capacity* of the fluid away from the surface is increased by the roughness-generated turbulence, heat cannot in fact be transferred at a rate greater than conduction into the surface will allow; no such restriction on the shear stress and skin friction, freed from a dependence on molecular transport properties if the roughness is large enough, exists.

An immediate consequence of this elementary argument is that the rate of heat transfer must be a function of both Reynolds number and Prandtl number in as much as they reflect a dependence on molecular transport properties.

Our object here is to construct a model of the flow adjacent to the roughened surface which, taken in conjunction with the Reynolds analogy between temperature and velocity distributions in the outer regions of the fluid, may be used to deduce from existing experimental data the form of the Reynolds number and Prandtl number dependence. In this way it is hoped to provide a basis for the analysis of future experiments on particular surfaces and for their extrapolation with respect to Reynolds number and Prandtl number. Secondarily and less precisely, the model leads to an expression for the rate of heat transfer which, with a constant adjusted to give a coarse fit to the existing data, may serve to provide an estimate when no specific information is available from experiment.

2. Model of the flow near the surface

In postulating a structure of the flow adjacent to the surface we shall confine attention to roughness which is so large that any notion of the existence of a viscous sublayer in its ordinary sense must be abandoned, a condition which is satisfied if $u_\tau h/\nu > 100$, where u_τ is the friction velocity, h the equivalent sand roughness height and ν the kinematic viscosity.

It will further be supposed that the roughness is closely spaced so that the shear stress carried by the fluid near the wall and with it the mean velocity in the stream direction fall rapidly as the troughs of the roughness are approached: for the gradient in shear stress balances the average resistance per unit volume of fluid offered by the roughness elements. (In the other extreme of 'widely spaced' roughness, not considered here, the main stream of fluid penetrates the gaps between the excrescences and is able to transmit its shear stress directly to part of the surface.) We may therefore regard the roughness as exposed to a highly sheared mean flow in which the streamwise velocity changes by an amount of order u_τ in a distance of order h.

The well-known characteristic of such a shear flow is the development of stream-wise vorticity largely concentrated in horseshoe eddies which wrap themselves around the individual excrescences and trail downstream, as roughly sketched in figure 1. The effect of these eddies is to draw fluid down into the valley-like regions between adjacent roughness elements which the fluid then scours before returning to mix with the main flow near the height of the roughness crests. It is suggested that the scouring action forms the basic convective mechanism of heat transfer at the wall. In this respect we might note that in the vicinity of the roughness troughs where the mean stream-wise velocity is evanescent the flow is predominantly that due to the eddy system, whereas near the crests there is a comparatively vigorous stream-wise motion available for receiving and rapidly dispersing any temperature difference imparted to it by the eddies.

Our model is then a very simple one.† It visualizes a kind of shallow sublayer

† A similar model, but manipulated in a different way and using a two-dimensional cavity-like flow as the basic eddy structure, has been proposed in a doctoral thesis by Dipprey (1961) whose work the authors became aware of only after the preparation of this paper had begun.

of fluid whose thickness is of order h in the deepest parts of which the motion is contributed by horseshoe eddies elongated in directions parallel to the wall and threaded between the roughness elements. The eddies are fed with vorticity from the stream-wise flow away from the roughness troughs and are subjected to frictional resistance at the solid surface where heat transfer between the fluid and wall occurs through conduction. The heat is communicated to the fluid in the space near and beyond the roughness crests, which may be part of a turbulent boundary layer or pipe or channel, by the convective motion of the horseshoe eddies although, as discussed in § 7, the fact that the eddies are turbulent (our attempt to give a simple description of them in terms of a quasi-steady flow conceals the fact that they are strictly an integral part of the turbulence†) strongly influences the rate at which they can receive heat from or donate heat to the wall, especially when the Prandtl number is much different from unity.

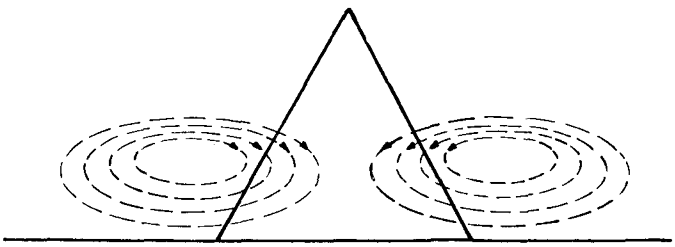

FIGURE 1. Rough sketch of the horseshoe eddies behind an excrescence.

Regarding the eddies as deeply embedded within the roughnesses does not destroy the idea that they are in major part responsible for transporting heat between the fluid and wall; although the crests of the roughness may be immersed in a more rapidly moving stream, the area they present is small compared with the surface area swept by the sublayer.

Based on the picture of individual roughness elements, it can be argued that the three-dimensional character of the above model is not appropriate to the case of two-dimensional roughness formed by (closely spaced) humps or grooves running perpendicular to the main stream direction, where the flow might more nearly resemble that in a genuine cavity. The acceptance of a model of this latter kind for two-dimensional roughness would not affect our qualitative argument because it retains the general feature of a surface scour by eddies, in this case lying mainly in a direction parallel to the roughness generators, although it has to be borne in mind that near the roughness crests turbulent velocity fluctuations, which are themselves of order u_τ, introduce a powerful element of three-dimensionality into the flow such as to impart to the cavity eddies components of circulation in the stream-wise direction. However, the scouring action would be performed principally by the circulatory components transverse to the main flow and we admit the possibility that the rate of heat transfer could differ quantitatively between roughness of two- and three-dimensional shapes (yet

† Relative to the fixed surface, the eddies form a group with distinct dimensions which fluctuate in position in response to the turbulent velocites near the roughness crests and whose mean position, strength and direction of circulation are subject to a similar lack of uniformity to that of the roughness itself.

the experiments analysed in § 6 reveal no systematic difference); for that matter, since the details of the flow in the sublayer and the surface which it scours must depend on the nature of the roughness we could not expect to find a universal relation between the heat transfer, Reynolds number and Prandtl number, but we suggest that its *form* should be similar for all kinds of closely packed roughness.

3. The rate of heat transfer across the sublayer

The mean temperature in the fluid at the outer edge of the sublayer is taken to be T_h and the temperature of the solid surface T_0. Since the characteristic thickness of the sublayer is h and the characteristic velocity of the eddying fluid in it is u_τ, the rate of heat transfer across unit area of the layer may be written

$$Q = \rho c u_\tau (T_h - T_0)\, B, \tag{1}$$

where ρ is the density of the fluid, c is its specific heat and B is a Stanton number which is a function of the local Reynolds number $u_\tau h/\nu$ and of the Prandtl number σ. Q may also be identified with the rate of heat transfer across unit area of the wall projected into a surface parallel to the main stream.

A plausible expression for B is

$$B = \frac{1}{\alpha}\left(\frac{u_\tau h}{\nu}\right)^{-m} \sigma^{-n}, \tag{2}$$

where α is a constant for a particular roughness. The values of m and n depend on the diffusivities in the sublayer; if purely molecular, it may be expected that $m \approx \frac{1}{2}$, $n \approx \frac{2}{3}$, whereas the effect of a turbulent contribution would be to decrease m and increase n. Further discussion of this question is deferred until the analysis of the experimental data has been presented.

4. The rate of heat transfer for a pipe, channel, or flat plate

Suppose the inner wall of a pipe of circular cross-section is roughened such that the equivalent sand roughness height is h. By this it is meant that when an incompressible fluid flows steadily through a pipe of radius a with velocity U_m averaged over its cross-section, U_m is related to the friction velocity u_τ by

$$U_m/u_\tau = 8^{\frac{1}{2}}\{2\log_{10}(a/h) + 1{\cdot}74\},$$

provided that $u_\tau h/\nu > 100$, Schlichting (1955). h is comparable with, but not necessarily equal to, the actual height of the excrescences.

Applied to the flow in the central core of the pipe, where the velocity profile obeys the velocity-defect law, Reynolds analogy yields

$$\frac{T_1 - T}{T_\tau} = \frac{U_1 - U}{u_\tau} + \delta\left(\frac{r}{a}\right). \tag{3}$$

T_1 and U_1 are the mean temperature and velocity on the axis $r = 0$ and T_τ is the friction temperature $Q/\rho c u_\tau$, Squire (1953); $\delta(r/a)$ is a function, whose value when $r \approx a$ is $O(u_\tau/U_m)$, which expresses the small difference between the temperature and velocity profiles due to the fact that the temperature of the fluid varies along the pipe whilst the velocity does not.

Near the wall, (3) becomes

$$\frac{T_1 - T_h}{T_\tau} = \frac{U_1 - U_h}{u_\tau} + \delta(1)$$

which, eliminating T_h by means of (1), leads to

$$\frac{T_1 - T_0}{T_\tau} = \frac{U_1 - U_h}{u_\tau} + B^{-1} + \delta(1). \tag{4}$$

U_h is the (unknown) stream-wise velocity near the outer edge of the roughness sublayer.

We introduce a Stanton heat transfer coefficient defined by

$$K_s = \frac{Q}{\rho c U_m(T_m - T_0)} = \frac{T_\tau}{(T_m - T_0)}\frac{u_\tau}{U_m}, \tag{5}$$

where T_m is a temperature averaged across a section of the pipe. If it is defined as the 'mixing cup' temperature, as measured in most experiments,

$$T_m = \int_0^a UTr\,dr \Big/ \int_0^a Ur\,dr,$$

Squire (1953) has shown that

$$\frac{T_1 - T_m}{T_\tau} = \frac{U_1 - U_m}{u_\tau} - \delta_m + \delta(1). \tag{6}$$

Again, δ_m is a small correction arising from the intrinsic difference between the temperature and velocity profiles.

It follows from (4) and (6) that

$$\frac{T_m - T_0}{T_\tau} = \frac{U_m - U_h}{u_\tau} + B^{-1} + \delta_m$$

or,

$$K_s^{-1} = \frac{U_m}{u_\tau}\left\{\frac{U_m - U_h}{u_\tau} + B^{-1} + \delta_m\right\}. \tag{7}$$

δ_m can be shown to be proportional to u_τ/U_m if quantities $O(u_\tau/U_m)^2$ are neglected. Squire's numerical results, applicable to a pipe with uniform heat flux across its wall, are well represented by $\delta_m = 17{\cdot}8u_\tau/U_m$. The value of δ_m for a pipe with a wall maintained at a uniform temperature is somewhat different, but since the correction is in general small the distinction between the two cases is barely worth making in practice, at any rate for a roughened pipe.

A difficulty lies in assigning a value to U_h. Certainly, appeal to the Nikuradse velocity profile, $U/u_\tau = 2{\cdot}5\log(y/h) + 8{\cdot}5$, is unhelpful. In the first place, the origin of y, the distance from the 'wall', is uncertain; secondly, the boundary between the sublayer and the main stream is only vaguely defined, and its distance from the roughness troughs it likely to be a good deal less than h; thirdly, and most importantly, the logarithmic law is not valid within the valleys between the roughness peaks, nor indeed at all close to the roughness because, like the logarithmic profile for a smooth wall, it is simply the consequence of assuming a region of overlap between a 'law of the wall' and a 'velocity-defect law' and strictly applies to the flow only within this limited region. All that can be conjectured is that U_h is of order u_τ and is probably at most a small multiple of u_τ which depends on the details of the roughness. The actual value of U_h/u_τ for a particular kind of roughness may be found approximately from the experimental data; within the accuracy of both the data and the method of analysis it turns out that U_h/u_τ is indistinguishable from zero for almost all the shapes of roughness examined and we shall subsequently neglect it. Equation (7) then reads

$$K_s^{-1} = \frac{U_m}{u_\tau}\left\{\frac{U_m}{u_\tau}+\frac{1}{B}+17{\cdot}8\frac{u_\tau}{U_m}\right\}. \tag{8}$$

Similar expressions for the Stanton number can be found for a channel and for the flow over a flat plate. *In the case of a channel* of half-width a, the resistance is given by

$$U_m/u_\tau = 5{\cdot}75\log_{10}(a/h)+5{\cdot}94,$$

provided that $u_\tau h/\nu > 100$. Taking the velocity defect law to be

$$\frac{U_1-U}{u_\tau} = 2{\cdot}5\log\left(\frac{y-a}{a}\right),$$

where y is measured from the wall and U_1 is the velocity on $y = a$, a theoretical estimate gives $\delta_m = 12{\cdot}6u_\tau/U_m$. The Stanton number is then given by

$$K_s^{-1} = \frac{U_m}{u_\tau}\left\{\frac{U_m}{u_\tau}+\frac{1}{B}+12{\cdot}6\frac{u_\tau}{U_m}\right\}. \tag{9}$$

U_m is the velocity averaged across the channel and K_s is, as before, based on the difference between the wall temperature and the mixing-cup temperature

$$\int_0^a UT\,dy\bigg/\int_0^a U\,dy.$$

For a flat plate parallel to a stream of velocity U_1 and temperature T_1 the Stanton number $K_s = Q/\rho c U_1(T_1-T_0)$ is given by

$$K_s^{-1} = \frac{U_1}{u_\tau}\left(\frac{U_1}{u_\tau}+\frac{1}{B}\right). \tag{10}$$

5. Wind tunnel experiment

Given experimentally determined values of K_s and u_τ, the sublayer Stanton number can be derived from (8), (9) or (10) thereby enabling a test of the relation $B \propto (u_\tau h/\nu)^{-m}$ to be made for each Prandtl number and, if successful, the exponent m to be found.

The dependence of B on Prandtl number is harder to ascertain. Whilst copious measurements of the heat transfer across roughened surfaces have been made, only a few are relevant to the present analysis in so far as they satisfy the conditions of dense packing and $u_\tau h/\nu > 100$: and they only apply to air ($\sigma = 0{\cdot}72$) and to water at a temperature corresponding to a Prandtl number of about 7.†

† More recently, measurements in water covering several values of the Prandtl number have become available and are described in § 6.

In order to supplement the information it was decided to make measurements at an intermediate Prandtl number, preferably around 3.

The most convenient technique (with a wind tunnel available) is to observe the rate of mass transfer of a feebly volatile substance into an airstream, which has the advantage of simplicity and offers the possibility of securing the desired Prandtl number without the careful control of temperature that would be required in an experiment on the heat transfer to a liquid for which the Prandtl number is highly temperature-sensitive.

Camphor was chosen for the experiment because at ordinary temperatures it has a suitably small vapour pressure and, for sublimation into an airstream, the Prandtl number (ratio of kinematic viscosity to the coefficient of inter-diffusion) is 3·2.

The arrangement of the experiment was straightforward. A flat plate spanned the low-turbulence wind tunnel at the Mechanics of Fluids Laboratory, which has a working section 20 in. square and a maximum airspeed of 100 ft. $\sec^{-1}$, and sheets of commercial figured glass were placed on the plate to form rough surfaces. They were 26 in. long, 16 in. wide, bounded at their side edges by end-plates, and were cut so that a test-piece 3 in. square could be easily removed and replaced; it was equidistant from the side-edges of the sheets and 15 in. downstream from their leading edges which were made round by the addition of strips of wooden moulding. A flap attached to the trailing edge of the supporting plate could be adjusted to provide a uniform static pressure in the stream direction over the working surface of the model. Two types of glass were used: one was mottled by irregular pyramids in relief and the other reeded, with the generators of the reed running perpendicular to the stream to represent a two-dimensional roughness.

The entire surface of each glass sheet was sprayed with a solution of camphor in methylated spirit and, when dry, the test-piece was removed and weighed and then put back carefully so as to form a continuous surface with the rest of the glass (a small correction was applied to the weight to allow for the loss by free convection sustained during the time it took to replace the test-piece and adjust the neighbouring sections of glass—a matter of a few minutes). The wind tunnel was started rapidly and run for a sufficient time, between 5 and 15 min, to allow roughly 0·1 g of the camphor to sublime, during which the boundary layer was traversed by a comb of small Pitot tubes, 1 mm. outside diameter, at a station just downstream of the test-piece; in this way the friction velocity and equivalent sand roughness height of the glass, possibly modified by the presence of a thin layer of camphor, could be determined.

The sublayer Stanton number was found from the relation analogous to (10)

$$K_g^{-1} = (U_1/u_\tau)\{(U_1/u_\tau) + B^{-1}\}.$$

K_g is the mass-transfer coefficient $G/\rho U_1 \psi_s$, where G is the rate of sublimation per unit area projected on to a plane parallel to the plate and ψ_s is the (saturated) concentration of camphor vapour at the rough surface. ψ_s, which is strongly temperature-dependent, was obtained from tabulated values of the vapour pressure of camphor, such as those given in the *Handbook of Physics and Chemistry*, the temperature of the airstream having been noted during a run.

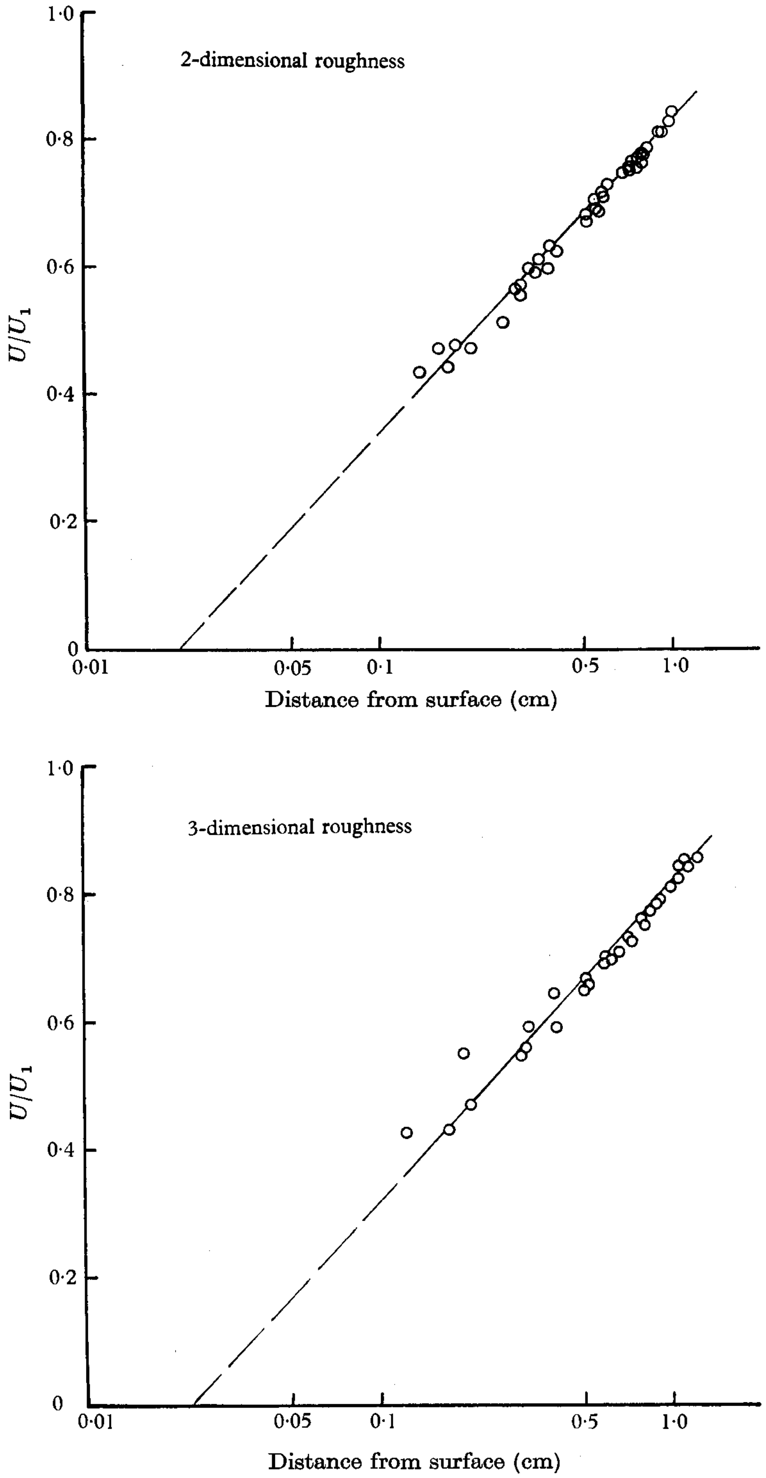

FIGURE 2. Boundary-layer traverses.

The results of the boundary-layer traverses are shown in figure 2 and cover a variety of windspeeds and air temperatures. The equivalent sand roughness height for the reeded plate is 6·3 mm and for the mottled plate is 6·9 mm. U_1/u_τ is 11·7 for both.

The values of $1/B$ deduced from the measured rates of mass transfer are given in table 1.

U_1 m sec^{-1}	Temp. °C	ψ_s $\times 10^3$	$u_\tau h/\nu$	K_g $\times 10^3$	B^{-1}
		Two-dimensional roughness			
25·0	21·0	1·68	896	1·64	40·3
18·0	21·5	1·74	644	1·84	34·7
6·1	17·0	1·26	218	2·76	19·3
15·3	18·0	1·36	547	2·36	30·6
23·2	18·5	1·41	832	1·84	34·7
18·3	25·8	2·36	656	1·98	31·3
10·3	23·2	1·98	368	2·38	24·3
13·1	20·8	1·68	470	2·14	28·2
9·0	21·6	1·77	323	2·46	23·0
7·9	22·8	1·93	283	2·54	21·9
		Three-dimensional roughness			
33·6	20·5	1·66	1314	1·46	46·9
23·2	18·0	1·51	907	1·42	48·5
28·6	18·5	1·44	1118	1·30	53·9
12·2	18·5	1·44	477	1·64	40·3
7·9	18·5	1·44	309	2·14	28·2
15·5	18·5	1·44	606	1·70	38·5
25·4	24·0	2·09	993	1·52	44·7
39·7	18·0	1·51	1565	1·23	57·6
20·8	17·0	1·27	774	1·56	43·1

TABLE 1. Results of the mass-transfer experiment.

6. The sublayer Stanton number B: analysis of experimental data

The data available for analysis which satisfy the condition $u_\tau h/\nu > 100$ and apply to densely distributed roughness can be summarized as follows.

Nunner (1956); air flow through pipes roughened by circumferential rings: $\sigma = 0{\cdot}72$ (arrangements with several widths and spacings of the rings were tested but only two sets of measurements, those for which the roughness may be described as densely packed, are useful here).

Pinkel (1954); air flow through a pipe roughened by helical rings: $\sigma = 0{\cdot}72$.

Lancet (1959); air flow through a channel with regular pyramidal roughness: $\sigma = 0{\cdot}72$.

Cope (1941); water flow through pipes with regular pyramidal roughness: $\sigma \approx 7{\cdot}5$.

To these can be added the sublimation experiments on flat surfaces with two- and three-dimensional roughnesses at a Prandtl number of 3·2 described herein.

Very recently some detailed and delicately executed heat transfer measurements by Dipprey (1961) at the California Institute of Technology came to the authors' notice.† They were made on the flow of water at a number of different temperatures through tubes indented by sand roughness. Dipprey's work is especially significant for it not only covers experimentally in a systematic way the variation of heat transfer with Reynolds number and Prandtl number but puts forward a theoretical model of the sublayer which resembles the one described in this paper. The essential difference between Dipprey's model and our own is that he treats the interstitial flow between the roughnesses as a two-dimensional cavity flow slowed down by friction within a thin layer adjacent to the surface of the roughness and separated from the main flow by a mixing region, whereas we regard the flow in the sublayer as essentially three-dimensional. In this respect we consider Dipprey's model to be more appropriate to the widely spaced two-dimensional type of roughness favoured by nuclear engineers for heat exchangers. Also, he takes the stream-wise velocity at the outer edge of the sublayer (what we have called U_h) to be $8{\cdot}5u_\tau$, a suggestion which was rejected in § 4. Dipprey in effect expresses his results in terms of $B \propto (u_\tau h/\nu)^{-m}\,\sigma^{-n}$, but the inclusion of the term $8{\cdot}5u_\tau$ which would add $8{\cdot}5U_m/u_\tau$ to the left of (8) and (9) and $8{\cdot}5U_1/u_\tau$ to the left of (10) leads to values of m and n roughly one-half as large as ours.

All the measurements referred to above were analysed to give values of B, and figure 3 shows $\log_{10}(1/B)$ plotted against $\log_{10}(u_\tau h/\nu)$. The lines drawn through the experimental points have a slope of 0·45 selected to give the best fit to Nunner's extensive measurements; treated individually, straight lines through each group of experimental points would have slopes varying from about 0·42 to 0·48.

An attempt was made to discover whether any consistent value of U_h/u_τ could be inferred from the data. In the case of a pipe, for example, (7) shows that

$$\frac{1}{B}-\frac{U_h}{u_\tau}=\frac{u_\tau}{U_m}(K_s^{-1}-17{\cdot}8)-\frac{U_m}{u_\tau}.$$

Plots, not reproduced here, were made of the right-hand side of this equation, together with the corresponding expressions for a channel and flat plate, against powers of $u_\tau h/\nu$ varying from 0·4 to 0·5 to examine whether straight lines through the data converged to a point, or small region, as $u_\tau h/\nu$ approached zero. The most consistent behaviour was found when the exponent of $u_\tau h/\nu$ was 0·45 and here the region of convergence was so near the origin as to suggest that within the accuracy of the analysis U_h/u_τ could not be distinguished from zero, with the single exception of our measurements on a plate with reeded (two-dimensional) roughness for which U_h was about $3u_\tau$.

The strong dependence of B on σ is evident in figure 3 from the vertical spread of the data, the value of $1/B$ increasing with σ. The relation between B and σ is rendered clearer by figure 4 where $\log_{10}[B^{-1}(u_\tau h/\nu)^{-0{\cdot}45}]$ deduced from figure 3 is plotted against $\log_{10}\sigma$. With the exception of Cope's measurements, which are

† We are indebted to Drs Roshko and Sabersky for making Dippreyʼs thesis available to us.

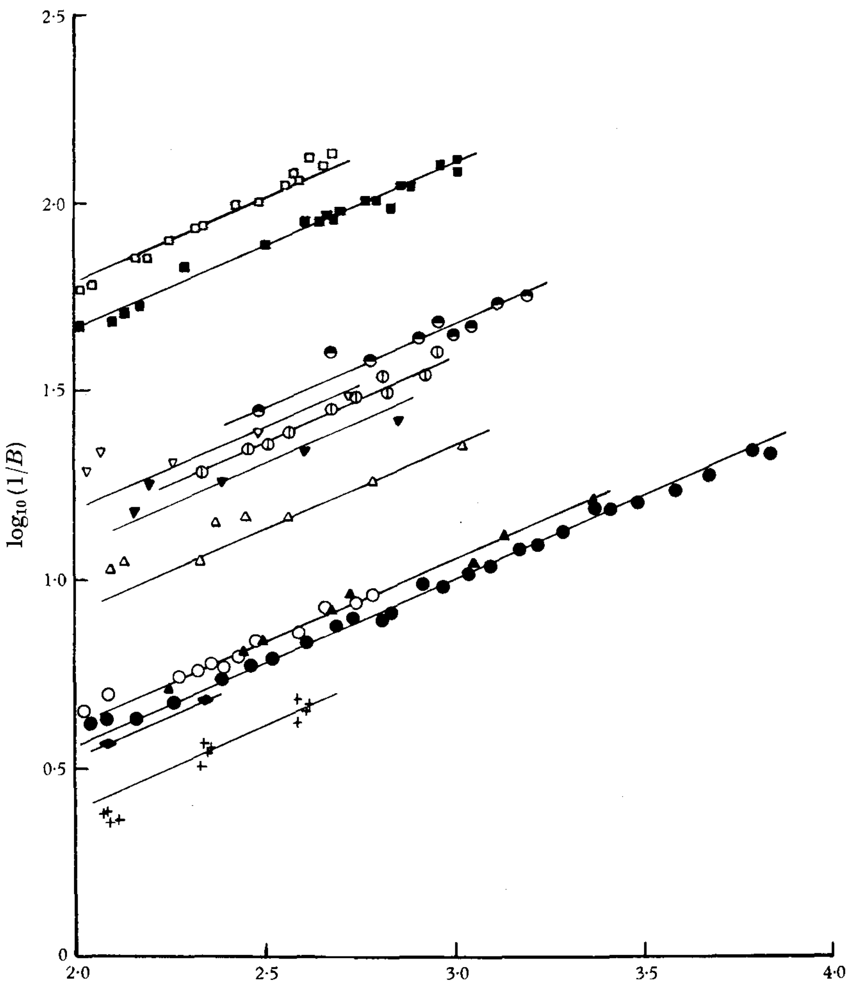

FIGURE 3. The sublayer Stanton number: Reynolds number dependence.

	Reference	Flow	Roughness type	σ
●	Nunner	Pipe	Circumferential rings	0·72
○	Nunner	Pipe	Circumferential rings	0·72
◆	Pinkel	Pipe	Helical rings	0·72
+	Lancet	Channel	Pyramids	0·72
■	Cope	Pipe	Pyramids	7·3
□	Cope	Pipe	Pyramids	7·5
⦶	Owen, Thomson	Plate	Spanwise humps	3·2
◓	Owen, Thomson	Plate	Irregular pyramids	3·2
▲	Dipprey	Pipe	Sand indentations	1·2
△	Dipprey	Pipe	Sand indentations	2·8
▼	Dipprey	Pipe	Sand indentations	4·4
▽	Dipprey	Pipe	Sand indentations	5·9

inexplicably high, the points fall within a band whose slope is 0·8. This suggests that (2) may be written

$$B = \frac{1}{\alpha}\left(\frac{u_\tau h}{\nu}\right)^{-0\cdot45} \sigma^{-0\cdot8}, \tag{11}$$

with α lying between 0·45 and 0·7.

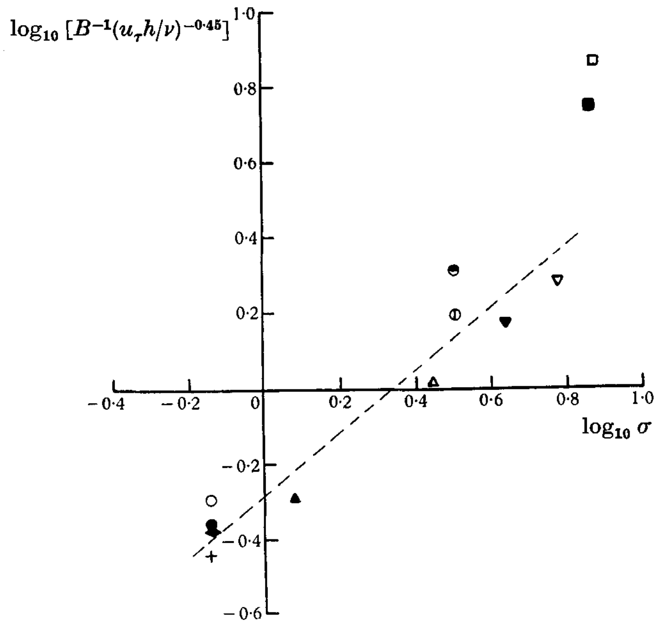

FIGURE 4. The sublayer Stanton number : Prandtl number dependence. See also caption to figure 3 for symbols.

The chain line in figure 4, for which $\alpha = 0\cdot52$, indicates the weighted mean of the band and its is suggested that in the absence of any guide from measurements on a particular surface this value of α may be used to provide an approximate estimate of the heat transfer rate. But in making this suggestion it must again be emphasized that our purpose has not been to establish a unique relation between B, $(u_\tau h/\nu)$ and σ—for we are convinced that one does not exist†—but to test whether the experimental data are consistent with the simple model proposed and, more particularly, with the expression (2) for the sublayer Stanton number B, at least over the limited range of the variables explored.

A further feature of figure 4 is the lack of any systematic difference in α between roughnesses of two- and three-dimensional character.

7. Interpretation of the form of the sublayer Stanton number

It remains to examine whether the relation $B \propto (u_\tau h/\nu)^{-0\cdot45} \sigma^{-0\cdot8}$ is physically consistent with the postulated model of the sublayer.

† Besides the influence of the roughness geometry on the fine structure of the flow in the sublayer, by defining the heat transfer coefficient with respect to unit area of the wall projected onto a surface parallel to the main stream we have glossed over the obvious effect of the roughness shape on the actual amount of surface exposed to the sublayer eddies.

As observed previously, the sublayer eddies are intrinsically part of the turbulent motion near the surface and may in fact be regarded as its energetic component. Fed with vorticity near the crests of the roughness, brought into being by strains imposed by the roughness elements, the eddies are retarded by friction at the solid surface. If it is assumed that the retardation is accomplished principally by viscous stresses it will be confined to a region adjacent to the surface of thickness $O(\nu h/u_\tau)^{\frac{1}{2}}$—a kind of (unsteady) viscous boundary layer embedded within the sublayer†—which, when the Prandtl number is unity, can also be identified as a thermal boundary layer. The corresponding rate of heat transfer is of order $\rho c\kappa \Delta T/\delta_T$, where κ is the thermometric conductivity, δ_T is the thickness of the surface layer and ΔT is the temperature difference between the wall and the fluid in the outer part of the sublayer. Recalling that the sublayer Stanton number B is defined as the rate of heat transfer across unit area parallel to the main stream divided by $\rho c u_\tau \Delta T$, it follows that $B \sim (u_\tau h/\nu)^{-\frac{1}{2}}$: approximately of the form established empirically in § 6.

To see what happens when the Prandtl number is significantly different from unity we shall consider the limiting case of $\sigma \gg 1$. Since this is equivalent to saying that $\kappa \ll \nu$, clearly the assumption that close to the surface of the roughnesses viscous stresses are much larger than Reynolds stresses does not include the implication that throughout the same region turbulent heat transport can be ignored in comparison with molecular transport: on the contrary, it is only within the deepest part of that region, *very* close to the surface, that the heat transfer may be regarded as purely conductive.

An estimate of the order of magnitude of the turbulent transport can be made by noting that adjacent to the surface the equations of motion and thermal energy reduce to

$$\nu \frac{\partial^2 v}{\partial y^2} = \frac{1}{\rho}\frac{\partial p}{\partial y}, \quad \frac{\partial^2 T}{\partial y^2} = 0,$$

where v, p the pressure, and T are instantaneous values and y and v are measured perpendicular to the surface. In casting the equations into the above simple form it is supposed that the dimension of the sublayer eddies in a direction perpendicular to the surface is considerably smaller than their dimensions parallel to the surface.

Using the continuity equation together with the conditions that the velocity and temperature fluctuations vanish at the solid, thermally conducting surface it is easy to show that the mean rate of turbulent heat transport is given by

$$\rho c\overline{vT} = \frac{c}{2\nu}\overline{\left(\frac{\partial p}{\partial y}\right)\left(\frac{\partial T}{\partial y}\right)}y^3 + \dots.$$

Since pressure and temperature fluctuations within the sublayer are determined by the sublayer eddies whose velocity, length and temperature scales are typically u_τ, h and ΔT we may write $\partial p/\partial y \sim \rho u_\tau^2/h$ and $\partial T/\partial y \sim \Delta T/h$, so that the rate of turbulent transport takes the form

$$\rho c\overline{vT} \sim \rho c u_\tau \Delta T (u_\tau h/\nu)(y/h)^3.$$

† The experiments of Roshko (1955) confirm this argument in the case of a two-dimensional cavity flow.

The rapid increase in $\rho c \overline{vT}$ with increasing distance from the surface suggests that appreciable temperature gradients can occur only within the region where the rate of heat transport is controlled mainly by molecular conductivity. The thickness δ_T of this thermal boundary layer is therefore defined in order of magnitude by

$$u_\tau \Delta T (u_\tau h/\nu)(\delta_T/h)^3 \sim \kappa(\Delta T/\delta_T).$$

It follows that

$$\sigma(u_\tau h/\nu)^2 (\delta_T/h)^4 \sim 1.$$

The Stanton number B for the sublayer is proportional to $\kappa/u_\tau \delta_T$; hence $B \sim (u_\tau h/\nu)^{-\frac{1}{2}} \sigma^{-\frac{3}{4}}$ which is close to the empirical expression $B \propto (u_\tau h/\nu)^{-0 \cdot 45} \sigma^{-0 \cdot 8}$.

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